Completing Inflation with Galilean Genesis

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arXiv:1501.xxxxx, T. Kobayashi, MY, J. Yokoyama

$$c = \hbar = 1, \quad M_G = 1/\sqrt{8\pi G} \sim 2.4 \times 10^{18}\mathrm{GeV}.$$

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Introduction

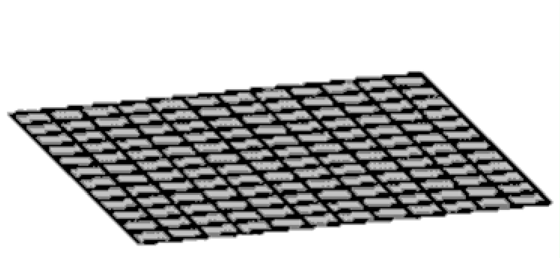
Inflation

Inflation, characterized as quasi De Sitter expansion, can naturally solve the problems of the standard big bang cosmology.

- **The horizon problem**
- **The flatness problem**
- **The origin of density fluctuations**
- **The monopole problem**
- …

Generic predictions of inflation

- Spatially flat universe



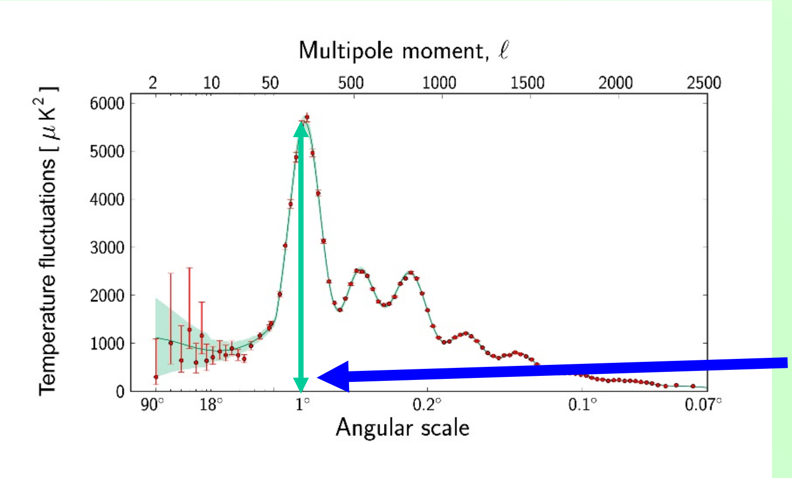
- Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations

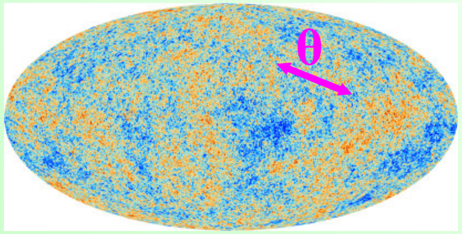
- Almost scale invariant and Gaussian primordial tensor fluctuations

→ Generates anisotropy of CMBR.

Observations of CMB anisotropies

Planck TT correlation :





Green line : prediction by inflation

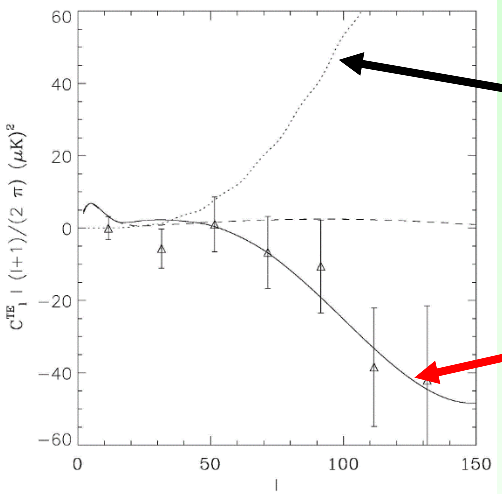
Red points : observation by PLANCK

Angle θ ~ 180° / l

Total energy density ←→ Geometry of our Universe

Our Universe is spatially flat !!

WMAP TE correlation :



Causal seed models

Superhorizon models

(adiabatic perturbations)

Unfortunately, primordial tensor perturbations have not yet been observed.

What happened before inflation ?

and/or

How did the Universe begin ?

Look back to the past of the Universe

It is often claimed that, if cosmic time goes back to the past, the energy density gets larger and larger, and it eventually reaches the Planck density.

So, unless one completes quantum gravity theory, one cannot discuss the state at the extremely early stage (or even at the onset) of the Universe.

For a perfect fluid : $T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - g_{\mu\nu} p$

The homogeneous and isotropic (Friedmann) Universe:

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$$

$$\Rightarrow \quad \dot{\rho} = -3H(\rho + p).$$

As long as $\rho + p \geq 0$ **(and H > 0 for the expanding Universe)**

$$\Rightarrow \quad \dot{\rho} \leq 0.$$

Null energy condition (NEC)

$$T_{\mu\nu}\xi^{\mu}\xi^{\nu} \geq 0$$

for any null vector ξ^{μ}.

$$(g_{\mu\nu}\xi^{\mu}\xi^{\nu} = 0)$$

This is the weakest among all of the local classical energy conditions.

For a perfect fluid : $T_{\mu\nu} = (\rho + p)\, u_{\mu}u_{\nu} - g_{\mu\nu}p$

$\Rightarrow$ **NEC** $\Leftrightarrow$ $\rho + p \geq 0$

As long as the NEC is conserved, the Universe cannot start from a low energy state.

How robust is the NEC ?

- **Canonical kinetic term with potential:**

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi).$$

$$\Rightarrow \left\{ \begin{array}{rcl} \rho & = & \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p & = & \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{array} \right. \Rightarrow \rho + p = \dot{\phi}^2 \geq 0.$$

(NEC is conserved)

- **How about k-inflation ?**

$$\mathcal{L} = K(\phi, X), \qquad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.$$

$$\Rightarrow \left\{ \begin{array}{rcl} \rho & = & 2XK_X - X \\ p & = & K \\ & & (K_X \equiv \partial K / \partial X) \end{array} \right. \Rightarrow \rho + p = 2XK_X.$$

Apparently, it looks that, if $K_X < 0$, it can violate the NEC.
But, this is not the case.

Primordial density fluctuations

Garriga & Mukhanov 1999

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2 e^{2\zeta} d\boldsymbol{x}^2$$

Comoving gauge :

$$\phi = \phi(t), \quad \delta\phi = 0.$$

Prescription:

- **Expand the action up to the second order**
- **Eliminate α and β by use of the constraint equations**
- **Obtain the quadratic action for ζ**

$$\Rightarrow \quad S_S^{(2)} = \int dt d^3x\, a^3\, M_G^2 \frac{\epsilon}{c_s^2}\left(\dot{\zeta}^2 - \frac{c_s^2}{a^2}\zeta_{,k}\zeta_{,k}\right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{XK_X}{M_{\rm pl}^2 H^2}, \qquad c_s^2 = \frac{K_X}{K_X + 2XK_{XX}}$$

(sound velocities of curvature perturbations)

In order to avoid the ghost and gradient instabilities, ε> 0 & c_s^2 > 0.

(Hsu et al. 2004)

$$\Rightarrow \quad \rho + p = 2XK_X > 0.$$

(See also Dubovsky et al. 2006)

Stable violation of the NEC

It is impossible to break the NEC stably within k-inflation.

$$\left(\mathcal{L} = K(\phi, X), \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.\right)$$

One may wonder how about introducing higher derivative terms.

Ostrogradski's theorem :

Assume that L = L(q, dot{q},ddot{q}) and $\frac{\partial L}{\partial \ddot{q}}$ **depends on ddot{q} :** **(Non-degeneracy)**

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\mathrm{d}^2}{\mathrm{d}^2 t}\left(\frac{\partial L}{\partial \ddot{q}}\right) = 0, \quad \Longrightarrow \quad q^{(4)} = q^{(4)}\left(q^{(3)}, \ddot{q}, \dot{q}, q\right).$$

This system always leads to ghost instabilities.

$$\frac{\partial L}{\partial \phi} - \partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \phi)}\right) + \partial_\mu \partial_\nu \left(\frac{\partial L}{\partial(\partial_\mu \partial_\nu \phi)}\right) = 0. \Longrightarrow \frac{i}{(p^2+m_1^2)(p^2+m_2^2)} = \frac{1}{m_2^2 - m_1^2}\left(\frac{i}{p^2+m_1^2} - \frac{i}{p^2+m_2^2}\right).$$

(propagators)

One loophole to introduce higher derivative terms is that equations of motion should be at most second order derivative ones.

Galileon

Nicolis et al. 2009
Deffayet et al. 2009

The theory has Galilean shift symmetry in flat space :

$$\phi \longrightarrow \phi + c + b_\mu x^\mu \qquad (\partial_\mu \phi \longrightarrow \partial_\mu \phi + b_\mu)$$

$$\begin{cases} \mathcal{L}_1 = \phi \\ \mathcal{L}_2 = (\partial\phi)^2 \\ \mathcal{L}_3 = (\partial\phi)^2 \, \Box\phi \\ \mathcal{L}_4 = (\partial\phi)^2 \left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] \\ \mathcal{L}_5 = (\partial\phi)^2 \left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3\right] \end{cases}$$

$$(\partial_\mu\partial_\nu\phi)^2 = \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi,$$
$$(\partial_\mu\partial_\nu\phi)^3 = \partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi$$

Lagrangian has higher order derivatives, but EOM is second order.

Is it possible to violate the NEC stably if one includes higher derivative terms ?

Galilean Genesis

Creminelli et al. 2010
Nicolis et al. 2009

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_G^2 R + f^2 e^{2\phi} (\partial\phi)^2 + \frac{f^3}{\Lambda^3} (\partial\phi)^2 \Box\phi + \frac{f^3}{2\Lambda^3} (\partial\phi)^4 \right]$$

(In the flat spacetime limit, this theory has conformal symmetry SO(4,2))

- **Energy-momentum tensor :**

$$\begin{cases} \rho = -f^2 \left(e^{2\phi} \dot\phi^2 - \frac{3}{2} \frac{f}{\Lambda^3} \dot\phi^4 - 6H \frac{f}{\Lambda^3} \dot\phi^3 \right), \\ p = -f^2 \left(e^{2\phi} \dot\phi^2 - \frac{1}{2} \frac{f}{\Lambda^3} \dot\phi^4 + 2 \frac{f}{\Lambda^3} \dot\phi^2 \ddot\phi \right). \end{cases}$$

- **A background solution, (t : -∞ -> 0)**

$$e^\phi \simeq \frac{1}{\sqrt{2Y_0}} \frac{1}{(-t)}, \qquad H \simeq \frac{h_0}{(-t)^3}, \qquad \left(a(t) \simeq 1 + \frac{h_0}{2(-t)^2} \right).$$

$$\left(Y_0 \equiv \frac{\Lambda^3}{3f}, \qquad h_0 \equiv \frac{1}{2M_G^2} \frac{f^3}{\Lambda^3} \right)$$

$$\Rightarrow \quad \rho + p \simeq -\frac{f^3}{\Lambda^3} \frac{4}{(-t)^4} < 0.$$

(Actually, you can verify that H increases.)

(The NEC is violated !!)

Primordial density fluctuations

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2 e^{2\zeta} d\boldsymbol{x}^2$$

Comoving gauge :

$$\phi = \phi(t), \quad \delta\phi = 0.$$

$$\Rightarrow \quad S_S^{(2)} = \int dt d^3x\, a^3 \left(\mathcal{G}_s \dot{\zeta}^2 - \frac{\mathcal{F}_s}{a^2} \zeta_{,k}\zeta_{,k} \right)$$

In order to avoid the ghost and gradient instabilities, Gs > 0 & Fs > 0.

$$\mathcal{G}_s = \mathcal{F}_s \simeq 6 M_G^4 \frac{\lambda^3}{f^3}(-t)^2 > 0.$$

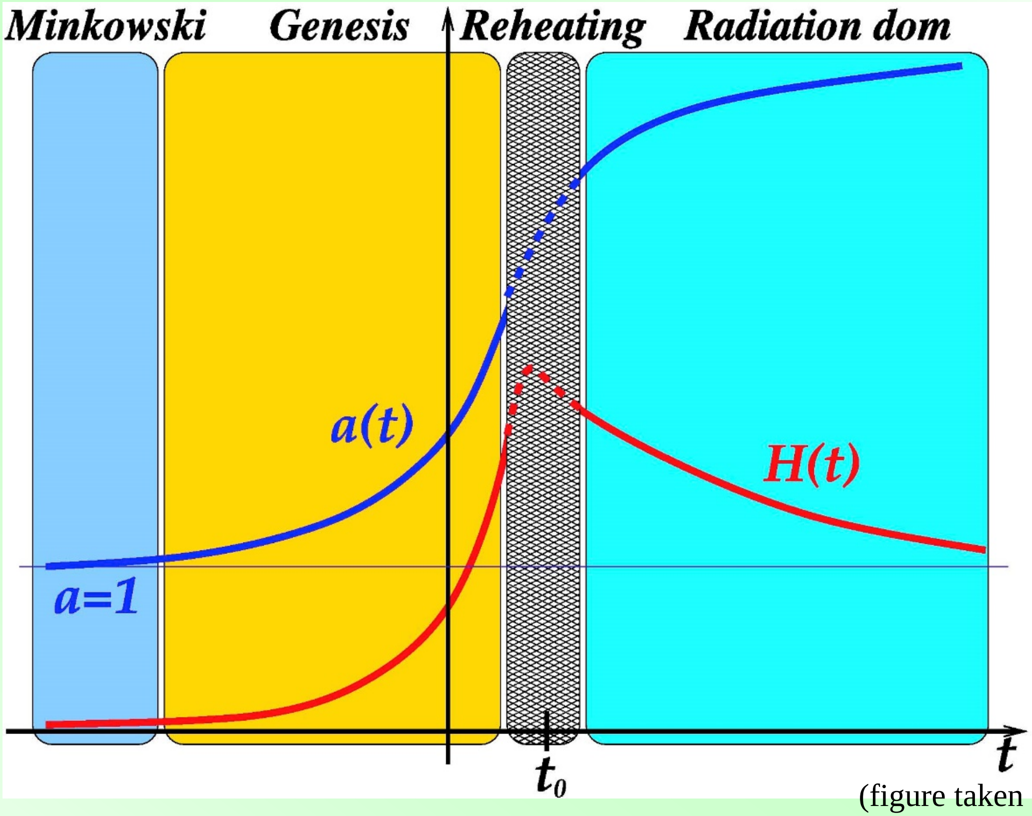
(The NEC is violated stably !!)

N.B.

- **A spectator field like curvaton is responsible for primordial density perturbations because the genesis field predicts too blue (ns ~3) perturbations in this simple model.**
- **Primordial tensor perturbations are not generated at first order.**

Galilean Genesis II

Creminelli et al. 2010
Nicolis et al. 2009



(figure taken from Creminelli et al. 1007.0027)

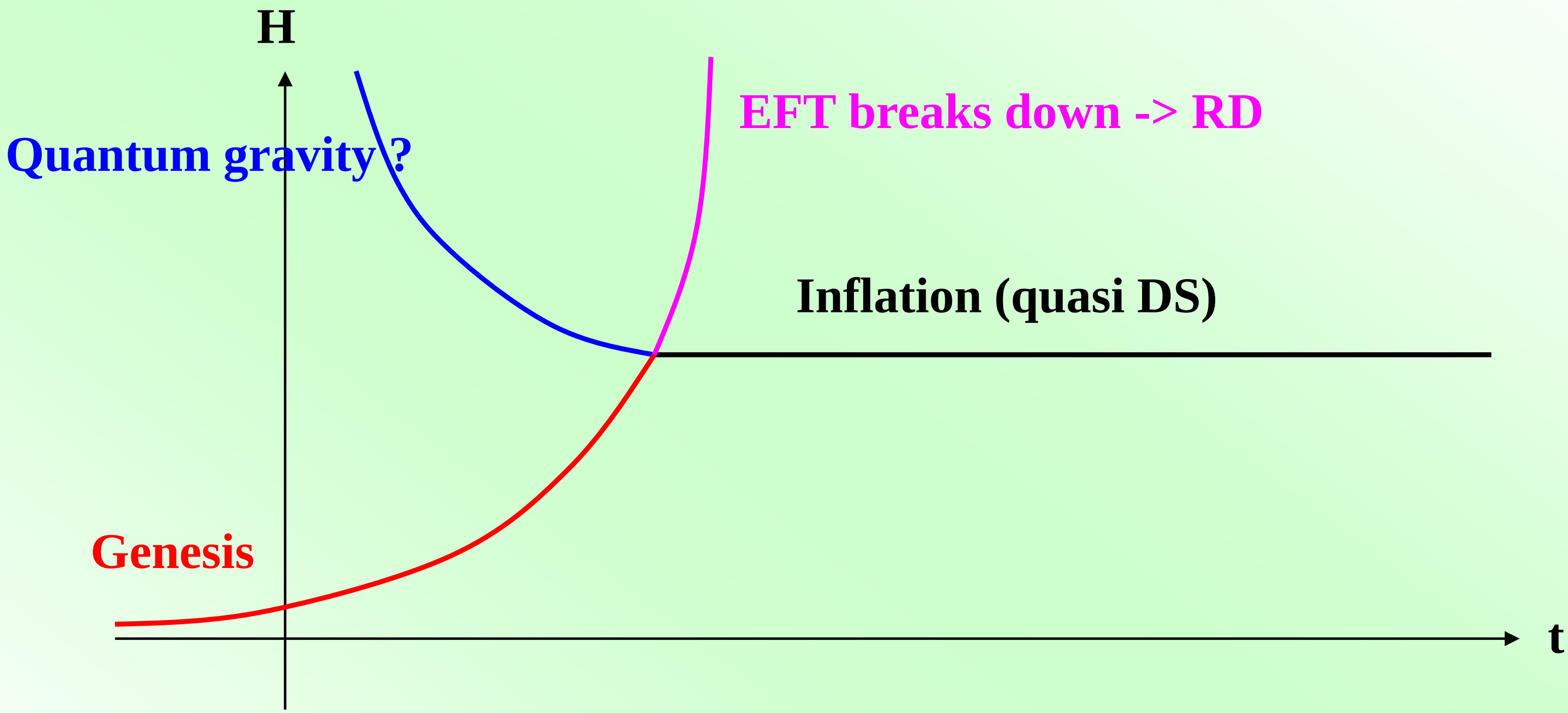
- **In this scenario, the effective theory breaks around $t \sim t_0 = 0$. So, it is assumed that the energy density of the genesis field is converted to radiation, in which hot Universe starts.**
- **Of course, this is not necessarily a fault of this scenario. A more fundamental theory will be able to describe the transition adequately.**

(See 1401.4024 written by Rubakov for good review)

From Genesis to inflation

From Genesis to inflation

Pirtskhalava et al. 2014



- **As a epoch before inflation (and the onset of the Universe), use of Galilean Genesis is proposed by Pirtskhalava et al.**
- **Unfortunately, in their concrete construction, the gradient instabilities appear during the transition from Genesis to inflation. They are dangerous for large k modes even during short period because of** $\propto e^{\mathrm{Im}(c_s)kt}$ **.**

Horndeski theory

Horndeski 1974
Deffayet et al. 2011
Kobayashi et al. 2011

Our concrete construction to realize such a scenario in a healthy way is based on the recent development beyond the Horndeski theory.

Horndeski theory (= Generalized Galileon) :

$$\begin{cases} \mathcal{L}_2 = \boxed{K(\phi, X)}, \\ \mathcal{L}_3 = -\boxed{G_3(\phi, X)}\Box\phi, \\ \mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = \boxed{G_5(\phi, X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$X = -\frac{1}{2}(\nabla\phi)^2, \qquad G_{iX} \equiv \partial G_i/\partial X.$$

This is the most general (single) scalar-tensor theory which yields second-order (scalar and gravitational) equations of motion.

But, in order to avoid the Ostrogradski instabilities, this requirement can be too strong. For this purpose, only time derivatives should be second order while spacial ones can be higher.

Galileon

Nicolis et al. 2009
Deffayet et al. 2009

The theory has Galilean shift symmetry in flat space :

$$\phi \longrightarrow \phi + c + b_\mu x^\mu \qquad (\partial_\mu \phi \longrightarrow \partial_\mu \phi + b_\mu)$$

$$\begin{cases} \mathcal{L}_1 = \phi \\ \mathcal{L}_2 = (\partial\phi)^2 \\ \mathcal{L}_3 = (\partial\phi)^2 \,\Box\phi \\ \mathcal{L}_4 = (\partial\phi)^2 \left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] \\ \mathcal{L}_5 = (\partial\phi)^2 \left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3\right] \end{cases}$$

$$(\partial_\mu\partial_\nu\phi)^2 = \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi,$$
$$(\partial_\mu\partial_\nu\phi)^3 = \partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi$$

Lagrangian has higher order derivatives, but EOM is second order.

Is it possible to violate the NEC stably if one includes higher derivative terms ?

Beyond Horndeski theory

Gleyzes et al. 2014
Gao 2014

ADM decomposition: $$ds^2 = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$
(φ= const surfaces)

$$\phi = \phi(t),\ \ X = \dot{\phi}^2(t)/(2N^2)$$

(φ and X are functions of t and N.)

Horndeski theory (= Generalized Galileon) :

$$\begin{cases} \mathcal{L}_2 &= K(\phi, X), \\ \mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & \quad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$\mathcal{L} = \sqrt{\gamma}N\sum_a L_a,$$

$$\begin{cases} L_2 = A_2(t, N), \\ L_3 = A_3(t, N)K, \\ L_4 = A_4(t, N)\left(K^2 - K_{ij}^2\right) + B_4(t, N)R^{(3)}, \\ L_5 = A_5(t, N)\left(K^3 - 3KK_{ij}^2 + 2K_{ij}^3\right) + B_5(t, N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right). \end{cases}$$

with $A_4 = -B_4 - N\frac{\partial B_4}{\partial N}, \quad A_5 = \frac{N}{6}\frac{\partial B_5}{\partial N}.$

Kij : extrinsic curvature
Rij(3) : intrinsic curvature

Gleyzes et al. (GLPV) pointed out that, even if the above relation is absent, the number of the propagating degrees of freedom remains unchanged.
Gao showed that further extension is possible.

Our setup

$$\mathcal{L} = \sqrt{\gamma} N \sum_a L_a$$

$$\begin{cases} L_2 = A_2(t,N), \\ L_3 = A_3(t,N)K, \\ L_4 = A_4(t,N)\left(\lambda_1 K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)}, \\ L_5 = A_5(t,N)\left(\lambda_2 K^3 - 3\lambda_3 K K_{ij}^2 + 2K_{ij}^3\right) \\ \qquad + B_5(t,N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2} g_{ij} R^{(3)}\right). \end{cases}$$

(The GLPV theory corresponds to the case with $\lambda_1 = \lambda_2 = \lambda_3 = 1$.)

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$\begin{cases} N = \overline{N}(t)\left(1 + \delta n\right), \\ N_i = \overline{N}(t)\partial_i \chi, \\ \gamma_{ij} = a^2(t) e^{2\zeta}\left(e^h\right)_{ij}. \end{cases}$$

curvature perturbations

tensor perturbations

$\left(h_{ii} = h_{ij,j} = 0\right)$

Perturbations

● **Tensor perturbations :**

$$\mathcal{L}_T^{(2)} = \frac{\overline{N}a^3}{8}\left[\frac{\mathcal{G}_T}{\overline{N}^2}\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial h_{ij})^2\right]$$

$$\begin{cases} \mathcal{G}_T &:= -2A_4 - 6(3\lambda_3 - 2)A_5 H, \\ \mathcal{F}_T &:= 2B_4 + \frac{1}{\overline{N}}\frac{dB_5}{dt}. \end{cases}$$

$$\left(H := \frac{\dot{a}}{(\overline{N}a)}\right)$$

● **Curvature perturbations :**

$$\mathcal{L}_S^{(2)} = \overline{N}a^3\left[\mathcal{G}_S\frac{\dot{\zeta}^2}{\overline{N}^2} + \zeta\left(\mathcal{F}_S\frac{\partial^2}{a^2} - \mathcal{H}_S\frac{\partial^4}{a^4}\right)\zeta\right] \Longrightarrow \omega^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S}k^2 + \frac{\mathcal{H}_S}{\mathcal{G}_S}\frac{k^4}{a^2}.$$

$$\begin{cases} \mathcal{G}_S &:= \frac{\Sigma\mathcal{G}_T^2}{\Theta^2 + \Sigma\mathcal{C}} + 3\mathcal{G}_T, \\ \mathcal{F}_S &:= \frac{1}{\overline{N}a}\frac{d}{dt}\left(\frac{a\Theta\mathcal{G}_B\mathcal{G}_T}{\Theta^2 + \Sigma\mathcal{C}}\right) - \mathcal{F}_T, \\ \mathcal{H}_S &:= \frac{\mathcal{G}_B^2\mathcal{C}}{\Theta^2 + \Sigma\mathcal{C}}. \end{cases}$$

$$\begin{cases} \Sigma &:= \overline{N}A_2' + \frac{1}{2}\overline{N}^2A_2'' + \frac{3}{2}\overline{N}^2A_3''H \\ &\quad +3\eta_4\left(2A_4 - 2\overline{N}A_4' + \overline{N}^2A_4''\right)H^2 \\ &\quad +3\eta_5\left(6A_5 - 4\overline{N}A_5' + \overline{N}^2A_5''\right)H^3, \\ \Theta &:= \frac{\overline{N}A_3'}{2} - 2\eta_4\left(A_4 - \overline{N}A_4'\right)H \\ &\quad -3\eta_5\left(2A_5 - \overline{N}A_5'\right)H^2, \\ \mathcal{G}_A &:= -2\eta_4A_4 - 6\eta_5A_5H, \\ \mathcal{G}_B &:= 2\left(B_4 + \overline{N}B_4'\right) - H\overline{N}B_5', \\ \mathcal{C} &:= (1-\lambda_1)A_4 - (6 + 9\lambda_2 - 15\lambda_3)A_5H. \end{cases}$$

$$(\eta_4 := (3\lambda_1 - 1)/2,\ \eta_5 := (9\lambda_2 - 9\lambda_3 + 2)/2)$$

N.B. ● **C = 0 for λ1 = λ2 = λ3 = 1.**

● **Even if Fs < 0 (with Gs > 0), the curvature perturbations with large k are stabilized for Hs > 0.**

Concrete example

$$\begin{cases} A_2 &= M_2^4 f^{-2(\alpha+1)}(t) a_2(N), \\ A_3 &= M_3^3 f^{-(2\alpha+1)}(t) a_3(N), \\ A_4 &= -\frac{M_G^2}{2} + M_4^2 f^{-2\alpha}(t) a_4(N), \\ A_5 &= M_5 f(t) a_5(N), \end{cases} \qquad (\alpha > 0)$$

$$\begin{cases} L_2 = A_2(t,N), \\ L_3 = A_3(t,N)K, \\ L_4 = A_4(t,N)\left(\lambda_1 K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)}, \\ L_5 = A_5(t,N)\left(\lambda_2 K^3 - 3\lambda_3 K K_{ij}^2 + 2K_{ij}^3\right) \\ \qquad +B_5(t,N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right). \end{cases}$$

Background dynamics : $\mathcal{L}^{(0)} = \bar{N} a^3 \left(A_2 + 3A_3 H + 6\eta_4 A_4 H^2 + 6\eta_5 A_5 H^3\right).$

$$\begin{cases} -\mathcal{E} &:= (\overline{N}A_2)' + 3\overline{N}A_3'H + 6\eta_4\overline{N}^2(\overline{N}^{-1}A_4)'H^2 + 6\eta_5\overline{N}^3(\overline{N}^{-2}A_5)'H^3 = 0, \\ \mathcal{P} &:= A_2 - 6\eta_4 A_4 H^2 - 12\eta_5 A_5 H^3 - \frac{1}{\overline{N}}\frac{d}{dt}\left(A_3 + 4\eta_4 A_4 H + 6\eta_5 A_5 H^2\right) = 0. \end{cases}$$

$$\left(' := d/d\overline{N}\right)$$

- **Genesis phase (t < t0) :** $f(t) \simeq \dot{f}_0 t \quad (\dot{f}_0 = \mathrm{const} < 0)$

$$\Rightarrow \begin{cases} \overline{N} \simeq N_0 \,(= \mathrm{const}) \quad \text{with} \quad a_2(N_0) + N_0 a_2'(N_0) = 0. \\ H = -\frac{\hat{p}}{2(2\alpha+1)\eta_4 M_G^2}\frac{N_0}{|\dot{f}_0|} f^{-(2\alpha+1)} \sim \frac{1}{(-t)^{2\alpha+1}}, \\ a = 1 - \frac{\hat{p}}{4\alpha(2\alpha+1)\eta_4 M_G^2}\frac{N_0^2}{\dot{f}_0^2} f^{-2\alpha}. \end{cases}$$

$$\left(\hat{p} = M_2^4 a_2(N_0) + (2\alpha+1)M_3^3 a_3(N_0)\frac{\dot{f}_0}{N_0}\right)$$

(The background dynamics α = 1 coincides with that of the original Genesis model.)

Concrete example II

- **Inflationary phase (tend > t > t0) :** $f(t) \simeq f_1 \,(= \mathrm{const})$

$$\Rightarrow \begin{cases} \overline{N} \simeq N_{\mathrm{inf}} \,(= \mathrm{const}), \\ H \simeq H_{\mathrm{inf}} \,(= \mathrm{const}). \end{cases}$$

with

$$\begin{cases} -\mathcal{E} = (N_{\mathrm{inf}} A_2)' + 3N_{\mathrm{inf}} A_3' H_{\mathrm{inf}} + 6\eta_4 N_{\mathrm{inf}}^2 (N_{\mathrm{inf}}^{-1} A_4)' H_{\mathrm{inf}}^2 \\ \qquad +6\eta_5 N_{\mathrm{inf}}^3 (N_{\mathrm{inf}}^{-2} A_5)' H_{\mathrm{inf}}^3 = 0, \\ \mathcal{P} = A_2 - 6\eta_4 A_4 H_{\mathrm{inf}}^2 - 12\eta_5 A_5 H_{\mathrm{inf}}^3 = 0. \end{cases}$$

N.B. A weak time dependence of f(t) yields slight deviation from exact DS.

- **Graceful exit (t > tend) :** $f(t) \sim t^{1/(\alpha+1)}$

$$\Rightarrow \begin{cases} \overline{N} \simeq N_{\mathrm{e}} \,(= \mathrm{const}), \\ H^2 \sim 1/t^2 \sim f^{-2(\alpha+1)} \propto 1/a^m \quad \left(m := 3N_{\mathrm{e}} a_2' / (N_{\mathrm{e}} a_2)' > 0\right). \end{cases}$$

with

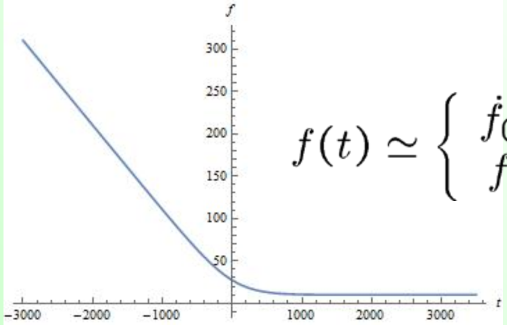
$$\begin{cases} -\mathcal{E} = (N_{\mathrm{e}} A_2)' + 3\eta_4 M_G^2 H^2 + \mathcal{O}(f^{-(3\alpha+2)}) = 0, \\ \mathcal{P} = A_2 + 3\eta_4 M_G^2 H^2 + \dfrac{2\eta_4 M_G^2}{N_{\mathrm{e}}} \dfrac{dH}{dt} + \mathcal{O}(f^{-(3\alpha+2)}) = 0. \end{cases}$$

Numerical calculations

From Genesis to inflation :

$$f(t) = \frac{\dot{f}_0}{2}\left\{t - \frac{\ln[2\cosh(st)]}{s}\right\} + f_1,$$

$$\dot{f}_0 = -10^{-1},\ f_1 = 10,\ s = 2\times 10^{-3} \simeq t_0^{-1}$$



$$f(t) \simeq \begin{cases} \dot{f}_0 t & \text{for} \quad t \ll t_0 \\ f_1 & \text{for} \quad t \gg t_0 \end{cases}$$

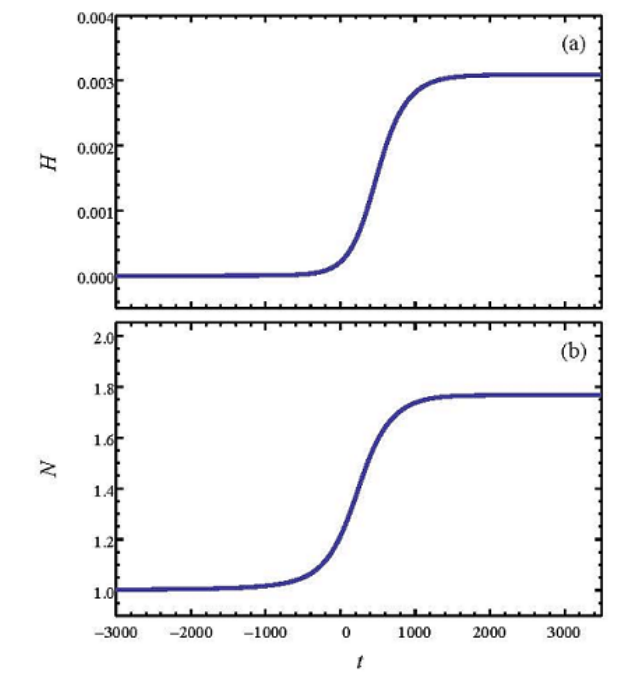


FIG. 2: The background evolution of the Hubble parameter H (a) and the lapse function N (b) around the genesis-de Sitter transition.

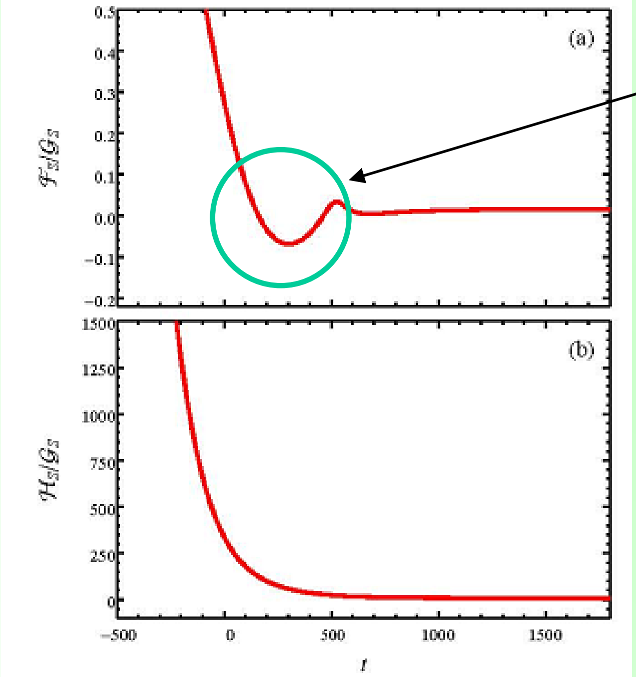


FIG. 3: The sound speed squared $\mathcal{F}_S/\mathcal{G}_S$ (a) and the coefficient of k^4 (divided by $\mathcal{G}_S$) (b) around the genesis-de Sitter transition.

During short period, Fs becomes negative. But, the perturbations for large k are stabilized thanks to the k^4 terms.

The perturbations for small k grow during the short period, but the growth is mild and finite.

Perturbations

- **Tensor perturbations :**

$$\mathcal{L}_T^{(2)} = \frac{\overline{N}a^3}{8}\left[\frac{\mathcal{G}_T}{\overline{N}^2}\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial h_{ij})^2\right]$$

$$\begin{cases} \mathcal{G}_T := -2A_4 - 6(3\lambda_3 - 2)A_5 H, \\ \mathcal{F}_T := 2B_4 + \frac{1}{\overline{N}}\frac{dB_5}{dt}. \end{cases}$$

$$\left(H := \frac{\dot{a}}{(\overline{N}a)}\right)$$

- **Curvature perturbations :**

$$\mathcal{L}_S^{(2)} = \overline{N}a^3\left[\mathcal{G}_S\frac{\dot{\zeta}^2}{\overline{N}^2} + \zeta\left(\mathcal{F}_S\frac{\partial^2}{a^2} - \mathcal{H}_S\frac{\partial^4}{a^4}\right)\zeta\right] \Longrightarrow \omega^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S}k^2 + \frac{\mathcal{H}_S}{\mathcal{G}_S}\frac{k^4}{a^2}.$$

$$\begin{cases} \mathcal{G}_S := \frac{\Sigma\mathcal{G}_T^2}{\Theta^2 + \Sigma\mathcal{C}} + 3\mathcal{G}_T, \\ \mathcal{F}_S := \frac{1}{\overline{N}a}\frac{d}{dt}\left(\frac{a\Theta\mathcal{G}_B\mathcal{G}_T}{\Theta^2 + \Sigma\mathcal{C}}\right) - \mathcal{F}_T, \\ \mathcal{H}_S := \frac{\mathcal{G}_B^2\mathcal{C}}{\Theta^2 + \Sigma\mathcal{C}}. \end{cases}$$

$$\begin{cases} \Sigma := \overline{N}A_2' + \frac{1}{2}\overline{N}^2A_2'' + \frac{3}{2}\overline{N}^2A_3''H \\ \quad +3\eta_4\left(2A_4 - 2\overline{N}A_4' + \overline{N}^2A_4''\right)H^2 \\ \quad +3\eta_5\left(6A_5 - 4\overline{N}A_5' + \overline{N}^2A_5''\right)H^3, \\ \Theta := \frac{\overline{N}A_3'}{2} - 2\eta_4\left(A_4 - \overline{N}A_4'\right)H \\ \quad -3\eta_5\left(2A_5 - \overline{N}A_5'\right)H^2, \\ \mathcal{G}_A := -2\eta_4A_4 - 6\eta_5A_5H, \\ \mathcal{G}_B := 2\left(B_4 + \overline{N}B_4'\right) - H\overline{N}B_5', \\ \mathcal{C} := (1-\lambda_1)A_4 - (6 + 9\lambda_2 - 15\lambda_3)A_5H. \end{cases}$$

$$(\eta_4 := (3\lambda_1 - 1)/2,\ \eta_5 := (9\lambda_2 - 9\lambda_3 + 2)/2)$$

N.B.

- **C = 0 for λ1 = λ2 = λ3 = 1.**
- **Even if Fs < 0 (with Gs > 0), the curvature perturbations with large k are stabilized for Hs > 0.**

Conclusions

- We constructed a concrete example from Galilean Genesis to inflationary phase followed by graceful exit, based on the recent development beyond the Horndeski theory.
- The sound velocities squared (or Fs) during the transition from Genesis to inflation becomes negative for a short period.
- But thanks to a non-trivial dispersion relation coming from the fourth order derivative term in the quadratic action, modes with higher k are completely stable and the growth of perturbations with smaller k is finite and controllable.
- Our model can describe a Genesis scenario with graceful exit (without inflationary phase), in which no (first order) primordial tensor perturbations are produced. The detection or non-detection of primordial tensor perturbations may discriminate Genesis scenarios with or without inflation.

Numerical calculations (parameters)

$$\begin{cases} L_2 = A_2(t,N), \\ L_3 = A_3(t,N)K, \\ L_4 = A_4(t,N)\left(\lambda_1 K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)}, \\ L_5 = A_5(t,N)\left(\lambda_2 K^3 - 3\lambda_3 K K_{ij}^2 + 2K_{ij}^3\right) \\ \qquad + B_5(t,N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right). \end{cases}$$

$$\begin{cases} A_2 = M_2^4 f^{-2(\alpha+1)}(t)a_2(N), \\ A_3 = M_3^3 f^{-(2\alpha+1)}(t)a_3(N), \\ A_4 = -\frac{M_G^2}{2} + M_4^2 f^{-2\alpha}(t)a_4(N), \\ A_5 = M_5 f(t)a_5(N), \end{cases}$$

From Genesis to inflation :

$$f(t) = \frac{\dot{f}_0}{2}\left\{t - \frac{\ln[2\cosh(st)]}{s}\right\} + f_1,$$

$$\dot{f}_0 = -10^{-1},\ f_1 = 10,\ s = 2\times 10^{-3} \simeq t_0^{-1}$$

$$a_2 = -\frac{1}{N^2} + \frac{N_0^2}{3N^4}, \quad a_3 = \frac{\gamma}{N^3}, \quad a_4 = a_5 = 0,$$

$$B_4 = M_G^2/2, \quad B_5 = 0.$$

$$M_G = M_2 = M_3 = 1,$$

$$\alpha = 1, \quad \lambda_1 = 1 + 10^{-3}, \quad N_0 = 1, \quad \gamma = 10.$$

Lagrangian

Why does Lagrangian generally depend on only
a position q and its velocity dot{q} ?

Newton recognized that an acceleration, which is given by the second time derivative of a position, is related to the Force :

$$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = \boldsymbol{F}\left(\boldsymbol{x}, \dot{\boldsymbol{x}}\right).$$

The Euler-Lagrange equation gives an equation of motion up to the second time derivative if a Lagrangian is given by L = L(q,dot{q},t).

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0, \quad \Longrightarrow \quad \ddot{q} = \ddot{q}\left(\dot{q}, q\right) \quad \Longrightarrow \quad q(t) = Q\left(\dot{q}_0, q_0, t\right).$$

(if $p := \frac{\partial L}{\partial \dot{q}}$ depends on dot{q} ⇔ non-degenerate condition.)

What happens if Lagrangian depends on higher derivative terms ?

Ostrogradski's theorem

Assume that L = L(q, dot{q},ddot{q}) and $\frac{\partial L}{\partial \ddot{q}}$ depends on ddot{q} :

(Non-degeneracy)

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\mathrm{d}^2}{\mathrm{d}^2 t}\left(\frac{\partial L}{\partial \ddot{q}}\right) = 0, \quad \Longrightarrow \quad q^{(4)} = q^{(4)}\left(q^{(3)}, \ddot{q}, \dot{q}, q\right).$$

Canonical variables :

$$\begin{cases} Q_1 := q, \quad P_1 := \dfrac{\partial L}{\partial \dot{q}} - \dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{\partial L}{\partial \ddot{q}}, \\ Q_2 := \dot{q}, \quad P_2 := \dfrac{\partial L}{\partial \ddot{q}}. \end{cases}$$

Non-degeneracy ⇔ there is a function a=a(Q_1,Q_2,P_2) such that $\left.\frac{\partial L}{\partial \ddot{q}}\right|_{q=Q_1,\dot{q}=Q_2,\ddot{q}=a} = P_2$.

Hamiltonian:

$$\begin{aligned} H(Q_1, Q_2, P_1, P_2) &:= P_1\dot{q} + P_2\ddot{q} - L \\ &= P_1 Q_2 + P_2 a(Q_1, Q_2, P_2) - L(Q_1, Q_2, a(Q_1, Q_2, P_2)). \end{aligned}$$

These canonical variables really satisfy the canonical EOM : $\dot{Q}_i = \frac{\partial H}{\partial P_i}, \quad \dot{P}_i = -\frac{\partial H}{\partial Q_i}.$

P_1 depends linearly on H so that no system of this form can be stable !!

From Genesis to inflation

Pirtskhalava et al. 2014

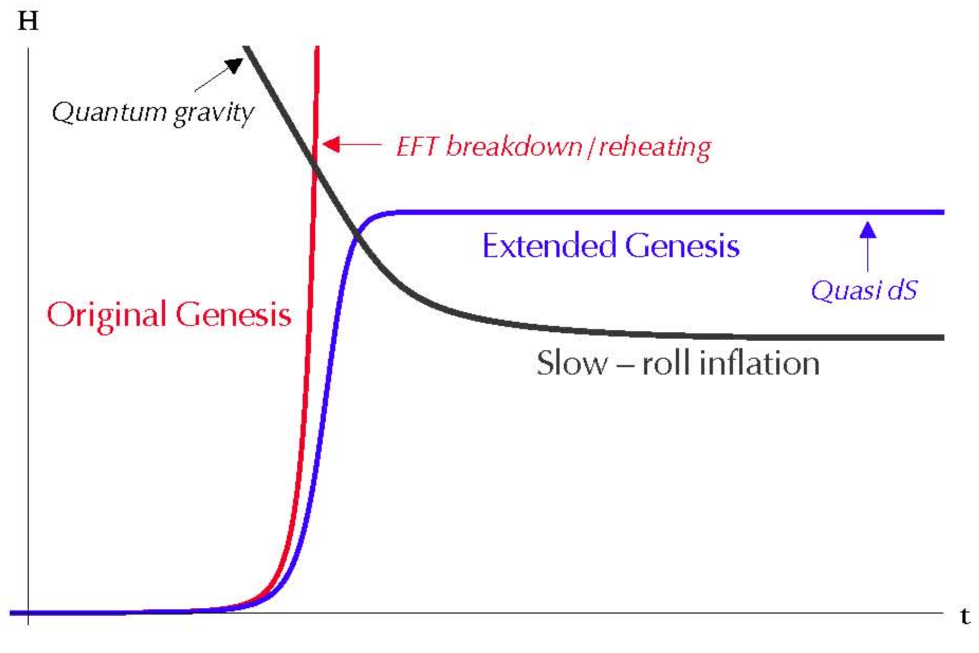


Figure 1: A sketch of the early universe's expansion rate as a function of time for the standard slow-roll inflation (black), as well as original (red) and extended (blue) genesis scenarios.

(figure taken from Pirtskhalava et al. 1410.0882)

- **As a epoch before inflation (and the onset of the Universe), use of Galilean Genesis is proposed by Pirtskhalava et al.**
- **Unfortunately, in their concrete construction, the gradient instabilities appear during a short period of the transition from Genesis to inflation.**

Numerical calculations

From Genesis to inflation :

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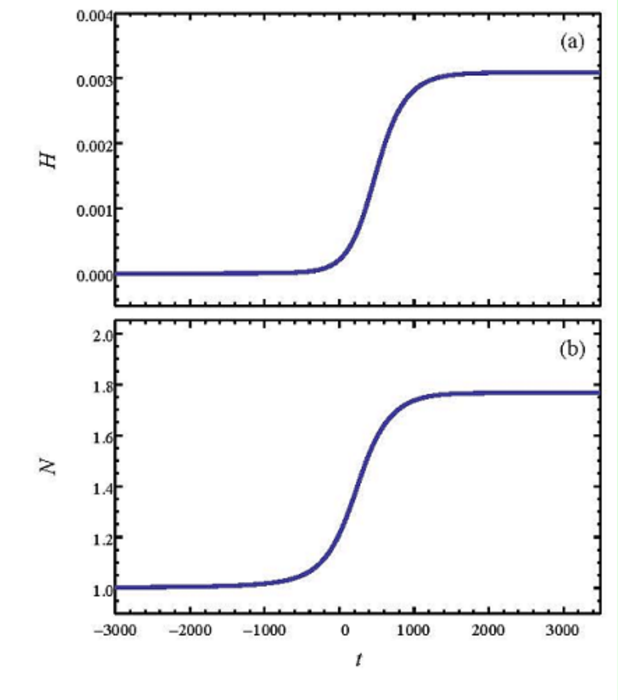
$$\dot{f}_0 = -10^{-1},\ f_1 = 10,\ s = 2\times 10^{-3} \simeq t_0^{-1}$$

$$\begin{cases} A_2 = M_2^4 f^{-2(\alpha+1)}(t) a_2(N), \\ A_3 = M_3^3 f^{-(2\alpha+1)}(t) a_3(N), \\ A_4 = -\dfrac{M_G^2}{2} + M_4^2 f^{-2\alpha}(t) a_4(N), \\ A_5 = M_5 f(t) a_5(N), \end{cases}$$



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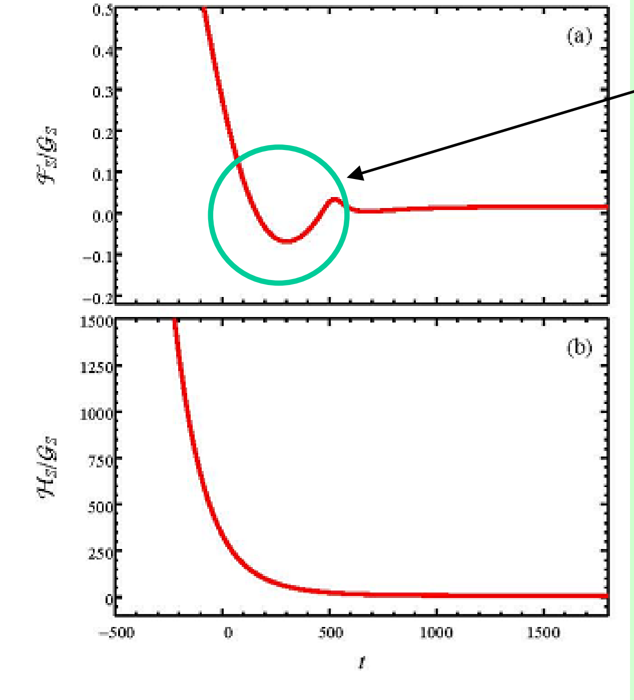

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